

To

Editorial Board Member

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Dear Prof Giorgio Grioli,

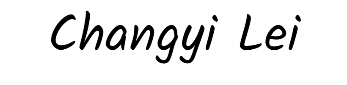
Re: Design and Stability Analysis of Semi-Implicit Cascaded Proportional-Derivative Controller for Underactuated Cart-Pole Inverted Pendulum System (ID: ROB-23-0030) by Changyi Lei, Ruobing Li, Quanmin Zhu

Many thanks for your instruction for the revision of this study. We have revised the paper in line with the editor and reviewer recommendations and corrections, and positively responded to all of the comments/questions raised. Please find the replies to the editor and the reviewers attached to the cover letter.

In addition, we have shown our gratitude to the editors and the anonymous reviewers for their helpful comments and constructive suggestions with regard to the revision of the paper in the Acknowledgements.

Should you have any further advice with regard to this paper, please contact us again.

Yours sincerely,



**Comment from Editor**

An extensive revision and rewriting is required to address the reviewers' concerns. In particular, the paper needs to be reorganized to give clear focus on the intended novel scientific contribution.

**General Response:**

We aim to use this general response to notify all reviewers and editor of the modification of our controller design. We revised our e3 in formula (25) from to . In this way, the controller is not only considering the stabilization of the pole, but also that of the cart to a given location . This modification makes our controller more powerful. Accordingly, most of the derivation process has been revised, and Jacobian matrix calculation has been again verified to ensure the validity of our calculations.

**Comments from Reviewer #1**

Overall this is a very interesting paper and is generally well-written.

However, there is, in my view, there is one major defect. The way it is written makes it unclear if it is developing a completely new cascade design method meant to be applicable to a broad range of systems, or if the results are limited to the cart-pole system.

\* The abstract makes it seem like it's limited to the cart-pole system.

\* The "contributions of this article" list make it seem like the results are general.

\* The discussion of underactuated systems make it seem like the results are supposed to be general.

\* The "workflow of our controller" section (lines 166-178) make it seem like it's fairly general.

\* The entire rest of the paper is limited to the cart-pole system, which makes it seem specific.

\* The title makes it seem specific.

If the results are intended to be applicable to systems other than the cart-pole system, then the authors need to add content giving the conditions a system must satisfy in order to apply this method. If the results are limited to the cart-pole system, then I think some of the language that makes the results seem more general needs to be modified to make the scope of the results very clear.

**Response**

We have modified all the above-mentioned sections of our paper in order to constrain the applicable range to cartpole system only. Specifically, "contributions of this article" is modified to focus on cartpole systems. We also shortened the “discussion of underactuated systems” in the introduction part. We also revise the “workflow of our controller”, by relating the “PD1”, “PD2”, “subplant1”, and “subplant2” description closely with the cartpole system from line 150-153. Thus we think the description here is now specific.

In addition, we also state in the last paragraph of our conclusion that this method has the potential to be extended to a class of underactuated systems.

My only other comment at this point is that I think the introduction is too comprehensive and reads like the introduction to a thesis. I found it interesting, actually, but a research paper should have a more concise and focused introduction that sets the context for the results in the paper.

**Response**

We have tried our best to delete some unnecessary introduction. In the meantime, we added some more discussions according to the comments from the other reviewer. So we personally think that maybe a comprehensive introduction is acceptable for a journal article?

**Comments from Reviewer #2**

1. There are some Typos along the paper.

**Response**

Sorry for our carelessness. We have revised as many typos as we can during our extensive re-writing process.

1. In the literature, some control algorithms about adaptive and robust stabilization applied to an underactuated robot must be considered. For instance:

'KL'-gain adaptation for attractive ellipsoid method. IMA Journal of Mathematical Control and Information, 32(3), 447-469.

Fuzzy logic and gradient descent-based optimal adaptive robust controller with inverted pendulum verification. Chaos, Solitons & Fractals, 151, 111257.

Adaptive reinforcement learning strategy with sliding mode control for unknown and disturbed wheeled inverted pendulum. International Journal of Control, Automation and Systems, 19(2), 1139-1150.

I suggest that a brief discussion of previous works on the introduction section can improve the underactuated pendular 2-degree of freedom systems preliminaries.

**Response**

Thank you for your recommendation. We added the last two papers along with a short discussion in line 43-49. However, we think the first paper is not specific on cartpole system. To avoid being too general on the description, we did not include the first paper you listed.

1. Please use another notation to define the mass of the pole. Note that this mass is defined as m and the notation of the International System of Units to define a distance is m.

**Response**

We now use for the cart mass, and for the pole mass.

1. The dynamic model is more realistic if the Coulomb friction is considered on this. Please consider this nonlinear friction type on the system dynamics.

**Response**

You are right, but please allow us to cope with the uncertainty issue in future research. As this paper is the first of our work in this direction, and that more robust controller will surely be developed based on this vanilla design in the future. We are also considering real experiments on our robotic platform, for which we have to consider the disturbance.

1. Please include some reference on the classical PD control considered on section 2.2.

**Response**

According to your suggestion, reference 6 and 34 have been added.

1. Before to design control scheme, the controllability of the system around the unstable equilibrium is required.

**Response**

Indeed, “Controllability Analysis Around Equilibrium Point” is added in section 3.1.

1. The proposed control scheme use a virtual control to drive some state variable as a desired set-point or trajectory. This procedure is commonly exposed on unmanned aerial vehicle flight, see for instance section 4 on the paper On the Finite Time Stabilization Via Robust Control for Uncertain Disturbed Systems. International Journal of Applied Mathematics and Computer Science, 33(1), 71-82. Please comment about main difference between both control schemes.

**Response**

We argue that the main difference lies in the implicit calculation of our proposed method. In equation (28) of the paper you mentioned, and also appear on both sides of the equation, similar to our situation. However, they did not solve this implicit equation directly. Instead, they used a bound to include to right side of the equation, and finally reaching approximate equation (33). We think this handling brings some conservatism, as all other robust controllers do. We can also see this point from the “≈” symbol of (33). On the contrary, our proposed method solves this implicit equation directly, which yield more precise control performance, but at the cost of tedious derivation process.

1. A control diagram block of the complete control scheme may help to understand the proposed algorithm.

**Response**

We have added Fig.3 to show the process of our control scheme.

1. Why the sampling period is required in (22)? Note that the study case is on continuous time system analysis. Please give some comments about this.

**Response**

This is a good detail. We have added Remark 4 to illustrate this handling, along with equation (25). The core idea of this handling is to transform a velocity target into a position target using this discrete approximation. In this way, the PD controller can be readily implemented in subplant2.

1. I think that some condition is required for Kp2 on (26). In this sense, what is the operating range for Kp2?

**Response**

See the following response.

1. Based on previous question, since the Jacobian matrix on (53) define the control algorithm stability. To guarantee the closed loop control system stability, please give some comments about the , Kp2 and Kd2.

**Response**

Beforehand, we want to clarify that is a function of Kp1, Kd1, so what you are referring to is the stability guarantee w.r.t the controller parameters.

Firstly, we admit that in this paper we cannot provide an analytical solution to the specific range of those parameters that stabilizes the system. We also illustrated this as a further research direction in conclusion. However, a critical advantage of our controller lies in similarly intuitive tuning process as conventional PID controller, which coincides with our statement of “intuitive to design” about the proposed method.

Secondly, using the derived Jacobian matrix, we can use whatever optimization method we have to find an approximate range of parameters that stabilizes the system by examining the eigenvalues of the resulting Jacobian matrix.

Last but not least, to further understand the influence of each parameter on the system performance, we implemented ablation study (around our chosen parameters) on both the baseline and proposed controller. The results (in appendix and Remark ) also show that our chosen parameters are Pareto optimal. I.E. by perturbing the current parameters, we cannot further improve the performance of x1 and x3 simultaneously. This is also important to ensure that the baseline method has achieved its optimal performance.

The above discussion is added as Remark 5 in line 301.

1. From my humble opinion, the stability analysis is based on closed loop controlled system linear dynamics. Thus, the Jacobian matrix is directly associated with the error function dynamics of the linear system. In this sense, a necessary condition to the stability is guarantee that based the so called Jacobian matrix be a Hurwitz matrix.

**Response**

We think you are referring to the Hurwitz stability theorem widely used in linear system design. We argue that Hurwitz stability theorem is targeted for matrix constructed using the coefficients of the characteristic equations of the system. However, we are using Jacobian matrix directly, which is different from the matrix that Hurwitz stability theorem focuses on. In addition, stability analysis based on Jacobian matrix is ubiquitous on the internet. Therefore, we believe our stability analysis is solid.

1. An interesting control problem should be to define the region of attraction of the closed loop controlled on the nonlinear system dynamics.

**Response**

We do not deny this is an interesting content. We can actually implement this result by illustrating a phase figure showing the convergence from many initial states. But after careful consideration, we think this paper may become a little messy if we further add this content. Note that we already added discussion of the parameter selection by implementing ablation studies (in appendix and remark 5), which shows the influence of each parameter on the performance, and that our chosen parameter is nearly Pareto optimal (we cannot further improve the performance of x1 and x3 simultaneously by devising the parameters anymore).

1. Gives more simulation information, for instance the software in which it is programed. For the use of OpenAI Gym, I suppose that the simulation is given on Python, but you need to define it.

**Response**

Thank you for pointing it out. We added a sentence to describe this simulation environment at the beginning of Section 4.1.

1. On the real time, the actuator is a DC-motor and it translate the voltage signal into a torque. I think that some remarks about this can be considered.

**Response**

We would like to put this to further research, where we plan to integrate methods from robust controller to cope with the uncertainties and noise. Since this paper is our first work in this direction, we hope to focus on the nominally theoretical part first.

1. Another fact on the real time test is the low amplitude random signal given by the electronics devices. So, in order to approximate to the real time implementation you can consider this effects on the signal measurements and the control output.

**Response**

Similar to previous comment, we would like to put this to further research. We are in the meantime considering applying our method to real robot, where we have to deal with the uncertainties.

1. I think that comparative study is well presented. However, this study must be improved by introduce absolute Error and mean absolute error (MAE) analysis.

**Response**

You are right. We have added MAE as well as other indices in section 4.3.3 for clearer comparison.

1. A comparative study with other approaches is mandatory.

**Response**

Yes. We implemented double-loop PD controller as baseline, described in section 4.2. Accordingly, section 4.3 has been revised extensively to show the comparison and analysis. This is a well-established PID-based method for cartpole system. Considering that our method is also a vanilla implementation of two (cascaded) PD controllers, we think it is convincible to compare with this baseline. We have also tuned the baseline carefully to ensure the validity of our results.

1. The authors should show the future research direction in conclusion.

**Response**

We have revised the last paragraph of the conclusion to include our outlook w.r.t this proposed method.